Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Influence of Extensibility on Tension in a Towed Cable

Alexander Simonenko*
Leningrad Shipbuilding Institute, Leningrad, USSR

TRANSPORTATION of cargo in barges, delivery of drilling platforms to the place of destination, and salvage operations are examples of the several operations performed with tug-boats using long cables. While towing in rough seas, the major part of tension in the cable is induced not because of the tug-boat thrust but because of the relative motions of the tug-boat and the towed body. To make these movements possible it is necessary either to increase the length of the cable or to increase its extensibility. Usually long steel cables are used with length over 3,000 ft. However, with such a long cable the maneuverability becomes rather poor. Sometimes it is better to insert a section of a more extensible cable in the steel towing cable, and thereby reduce the overall towline length. The aim of this article is to provide a formula that will make it possible to estimate the influence of extensibility on tension in the cable.

To derive equations of motion for a two-dimensional cable system, we make the following assumptions: 1) the equations of motion are derived in a fixed, orthogonal (x,y) coordinate system. The X-axis coincides with the calm water surface, and the Y-axis is downward. These coordinates are shown in Fig. 1; 2) the cable is fully submerged; and 3) a uniform stream does not influence the shape of the cable in a steady-state motion.

The forces acting on a cable element are shown on Fig. 2. Here

$$R(\dot{v}) = C_n \tfrac{1}{2}\,\rho \dot{v}^2 d_c \sin^2\beta ds$$

$$R(\dot{w}) = C_n \tfrac{1}{2}\rho \dot{w}^2 d_c \cos^2\beta ds$$

d_c is the cable diameter; q is the weight per unit length of the cable in water; C_n is the drag coefficient for flow normal to the cable; ρ is the fluid mass density; $\dot{v}$, $\dot{w}$ are velocities in the X and Y directions. The equations of motion in the X and Y directions are as follows:

X-direction

$$d(T\cos\beta) + c\cos^2\beta\sin\beta \left|\frac{\partial w}{\partial t}\right| \frac{\partial w}{\partial t} ds - c\sin^3\beta \left|\frac{\partial v}{\partial t}\right| \frac{\partial v}{\partial t} ds = m ds \frac{\partial^2 v}{\partial t^2} \quad (1)$$

Y-direction

$$d(T\sin\beta) - c\cos^3\beta \left|\frac{\partial w}{\partial t}\right| \frac{\partial w}{\partial t} ds + c\sin^2\beta\cos\beta \left|\frac{\partial v}{\partial t}\right| \frac{\partial v}{\partial t} ds + q ds = m ds \frac{\partial^2 w}{\partial t^2} \quad (2)$$

Received April 1, 1975; revision received April 10, 1975.
Index category: Marine Mooring Systems and Cable Mechanics.
*Associate Professor. Visiting Scholar, University of California, Berkeley.

where c is the modified drag coefficient $= (\rho/2) d_c C_n$ and m is the mass per unit length of the cable plus the added mass of water. The static pressure of the fluid is not taken in consideration (as it is in Ref. 1) because only nearly horizontal cables are considered here. These equations are not convenient because of the presence of the moduli $|\partial w/\partial t|$ and $|\partial v/\partial t|$. Consequently let us limit the time period to that when $v/t > 0$; $\partial w/\partial t < 0$. This is the period of time when the tension in the cable is increasing. For further considerations we make an assumption which is commonly used for towing of heavy objects: when $T/q \geq 2500$ m, provided that $T/qL \geq 2.5$, $\sin^n \beta = \beta$, $\cos\beta = 1$, $\sin^n\beta = 0$ when $n > 2$. In this case, Eqs. (1) and (2) simplify to the following:

$$\frac{\partial T}{\partial x} - T\beta \frac{\partial \beta}{\partial x} - m \frac{\partial^2 v}{\partial t^2} - c\beta\left(\frac{\partial w}{\partial t}\right)^2 = 0 \quad (3a)$$

$$\frac{\partial T}{\partial x}\beta + \frac{\partial \beta}{\partial x} T - m \frac{\partial^2 w}{\partial t^2} + c\left(\frac{\partial w}{\partial t}\right)^2 + q = 0 \quad (3b)$$

Denote by T_0 and β_0, respectively, the cable tension and the angle between the X direction and the tangent to the cable in its equilibrium position, and denote their change due to the cable movement by T_g and β_g. Then, taking into consideration that

$$\frac{\partial T_0}{\partial x} - T_0\beta_0 \frac{\partial \beta_0}{\partial x} = 0$$

$$\frac{\partial T_0}{\partial x}\beta_0 + T_0 \frac{\partial \beta_0}{\partial x} + q = 0,$$

we obtain from Eq. (3) two equations for T and β:

$$\frac{\partial T_g}{\partial x} - T_0\beta_g \frac{\partial \beta_0}{\partial x} - T_g\beta_0 \frac{\partial \beta_0}{\partial x} - T_g\beta_g \frac{\partial \beta_0}{\partial x} - T_0\beta_0 \frac{\partial \beta_g}{\partial x} - T_0\beta_g \frac{\partial \beta_g}{\partial x} - T_g\beta_0 \frac{\partial \beta_0}{\partial x} - T_g\beta_g \frac{\partial \beta_g}{\partial x} - m \frac{\partial^2 v}{\partial t^2} - c\beta_0\left(\frac{\partial w}{\partial t}\right)^2 - c\beta_g\left(\frac{\partial w}{\partial t}\right)^2 = 0 \quad (4a)$$

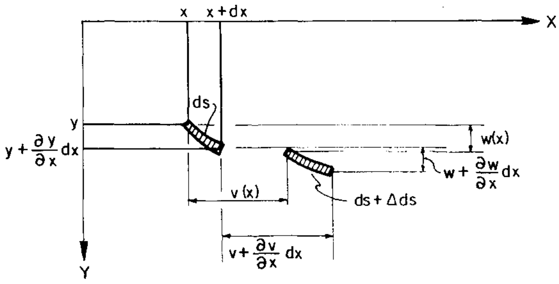

Fig. 1 Definition of lengths and coordinates used for calculating cable shape. ds = the length of an element of arc of the cable in its initial position; $ds + \Delta ds$ = the length of an element of arc of the cable after the form of the cable is changed; v = horizontal displacement of a cable point; and w = vertical displacement of a cable point.

$$\frac{\partial T_g}{\partial x}\beta_0+\frac{\partial T_g}{\partial x}\beta_g+\frac{\partial T_0}{\partial x}\beta_g+T_g\frac{\partial \beta_g}{\partial x}$$

$$+T_g\frac{\partial \beta_0}{\partial x}+T_g\frac{\partial \beta_g}{\partial x}-m\frac{\partial^2 \mathrm{w}}{\partial t^2}+c\left(\frac{\partial \mathrm{w}}{\partial t}\right)^2=0 \qquad (4b)$$

To find a connection between v and w we compare the length of the cable element in its initial position and after displacement:

$$(ds+\Delta ds)^2-ds^2=ds^2+2ds\Delta ds+\Delta ds^2-ds^2\approx 2ds\Delta ds$$

On the other hand, we can write this as follows:

$$\left(dx+\frac{\partial \mathrm{v}}{\partial x}ds\right)^2+\left(dy+\frac{\partial \mathrm{w}}{\partial x}dx\right)^2$$

$$-(dx^2+dy^2)\approx 2\frac{\partial \mathrm{v}}{\partial x}dx^2+2\frac{\partial \mathrm{w}}{\partial x}dxdy$$

Hence

$$ds\Delta ds=\frac{\partial \mathrm{v}}{\partial x}dx^2+\frac{\partial \mathrm{w}}{\partial x}dxdy \qquad (5)$$

The extension of the cable element is given by

$$\Delta ds=\frac{T_g}{\epsilon d_c^2}ds \qquad (6)$$

where ϵ is the coefficient of cable extensibility. After substituting Eq. (6) into Eq. (5), we obtain

$$\frac{\partial \mathrm{v}}{\partial x}=\frac{\partial y}{\partial x}\frac{\partial \mathrm{w}}{\partial x}+\frac{T_g}{\epsilon d_c^2}\left[1+\left(\frac{\partial y}{\partial x}\right)^2\right] \qquad (7)$$

It was assumed earlier that $\tan\beta_o=\beta_o$. Hence

$$T_g=\epsilon d_c^2\left[\frac{\partial \mathrm{v}}{\partial x}+\beta_o\frac{\partial \mathrm{w}}{\partial x}\right] \qquad (8)$$

It is well known that

$$T_o=T_h+\frac{q^2}{8T_h}(\ell-2x)^2 \qquad (9)$$

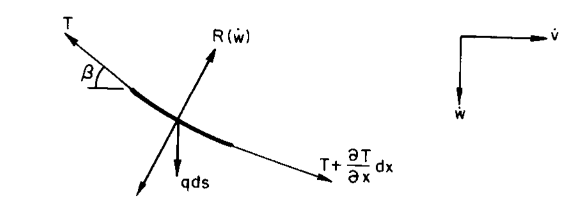

Fig. 2 Forces acting on a cable element of length *ds*.

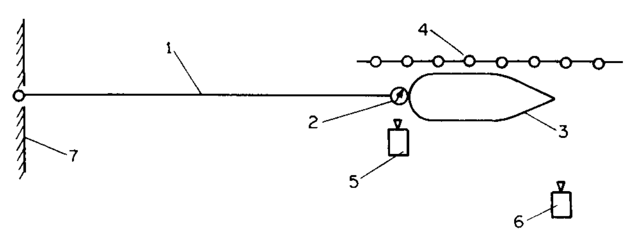

Fig. 3 Scheme of the experiment: 1) cable; 2) dynamometer; 3) tow-boat; 4) line of floats; 5,6) movie cameras; 7) pier.

where T_h is the thrust of the tow-boat and ℓ is the distance between the points where the cable is fastened.

Substitution of Eqs. (8) and (9) into Eq. (4) leads to the following system of nonlinear partial differential equations:

$$\epsilon d_c^2\left[\frac{\partial^2 \mathrm{v}}{\partial x^2}+\frac{\partial \beta_0}{\partial x}\frac{\partial \mathrm{w}}{\partial x}+\beta_0\frac{\partial^2 \mathrm{w}}{\partial x^2}\right]$$

$$-\left[T_h+\frac{q^2}{8T_h}(\ell-2x)^2\right]\beta_g\frac{\partial \beta_0}{\partial x}-\epsilon d_c^2\left[\frac{\partial \mathrm{v}}{\partial x}\right.$$

$$\left.+\beta_0\frac{\partial \mathrm{w}}{\partial x}\right]\beta_0\frac{\partial \beta_0}{\partial x}-\epsilon d_c^2\left[\frac{\partial \mathrm{v}}{\partial x}+\beta_0\frac{\partial \mathrm{w}}{\partial x}\right]\frac{\partial \beta_0}{\partial x}$$

$$-\left[T_h+\frac{q^2}{8T_h}(\ell-2x)^2\right]\beta_0\frac{\partial \beta_g}{\partial x}-\left[T_h\right.$$

$$\left.+\frac{q^2}{8T_h}(\ell-2x)^2\right]\beta_g\frac{\partial \beta_g}{\partial x}$$

$$-\epsilon d_c^2\left[\frac{\partial \mathrm{v}}{\partial x}+\beta_0\frac{\partial \mathrm{w}}{\partial x}\right]\beta_0\frac{\partial \beta_g}{\partial x}-\epsilon d_c^2\left[\frac{\partial \mathrm{v}}{\partial x}\right.$$

$$\left.+\beta_0\frac{\partial \mathrm{w}}{\partial x}\right]\beta_g\frac{\partial \beta_g}{\partial x}-m\frac{\partial^2 \mathrm{v}}{\partial t^2}+c\beta_0\left[\frac{\partial \mathrm{w}}{\partial t}\right]^2$$

$$+c\beta_g\left[\frac{\partial \mathrm{w}}{\partial t}\right]^2=0 \qquad (10a)$$

$$\epsilon d_c^2\left[\frac{\partial^2 \mathrm{v}}{\partial x^2}+\frac{\partial \beta_0}{\partial x}\frac{\partial \mathrm{w}}{\partial x}+\beta_0\frac{\partial^2 \mathrm{w}}{\partial x^2}\right]\beta_0$$

$$+\epsilon d_c^2\left[\frac{\partial^2 \mathrm{v}}{\partial x^2}+\frac{\partial \beta_0}{\partial x}\frac{\partial \mathrm{w}}{\partial x}+\beta_0\frac{\partial^2 \mathrm{w}}{\partial x^2}\right]\beta_g$$

$$+\frac{q^2}{8T_h}(\ell-2x)\beta_g+\left[T_h\right.$$

$$\left.+\frac{q^2}{8T_h}(\ell-2x)^2\right]\frac{\partial \beta_g}{\partial x}+\epsilon d_c^2\left[\frac{\partial \mathrm{v}}{\partial x}\right.$$

$$\left.+\beta_0\frac{\partial \mathrm{w}}{\partial x}\right]\frac{\partial \beta_0}{\partial x}+\epsilon d_c^2\left[\frac{\partial \mathrm{v}}{\partial x}+\beta_0\frac{\partial \mathrm{w}}{\partial x}\right]\frac{\partial \beta_g}{\partial x}$$

$$-m\frac{\partial^2 \mathrm{w}}{\partial t^2}+c\left[\frac{\partial \mathrm{w}}{\partial t}\right]^2=0 \qquad (10b)$$

Although these equations can be computerized they would require much computer time. To obtain approximate formulas for T_g, we assume that the cable elements will follow the same trajectories as those of an inextensible cable in air. The influence of water resistance will result in some change in the magnitude of the w displacement:

$$\mathrm{v}=3\alpha\left[\frac{x}{\ell}-\frac{2x^2}{\ell^2}+\frac{4}{3}\frac{x^3}{\ell^3}\right] \qquad (11a)$$

$$\mathrm{w}=A(xl-x^2) \qquad (11b)$$

where a is the displacement along the x-direction of the point where the cable is fastened, and A is a coefficient that does not depend upon the x-coordinate. Now it is sufficient to consider only one of Eqs. (10). In the steady-state position the cable form was

$$y=(q/2T_h)(\ell x-x^2) \qquad (12a)$$

If the left-hand point of the cable is fixed while the right-hand point moves with the displacement of a, b in the X and Y directions respectively, the boundary conditions are

$$x=0: \qquad v=0,\ w=0;$$

$$x=\ell: \qquad v=a(\cos\frac{2\pi t}{\tau}-1),\ w=b\sin\frac{2\pi t}{\tau} \tag{12b}$$

where τ is the period of the movement of the right-hand point. We can find the coefficient A in Eq. (11) by using the second of the Eqs. (10). In addition, we assume that vertical displacement of the right-hand point does not influence significantly the magnitude of A, i.e., $x=\ell$: $w=0$. Then Eq. (10) becomes

$$\begin{aligned}
&\epsilon d_c^2\Big[\frac{12a}{\ell^3}(\ell-2x)(\cos\frac{2\pi t}{\tau}-1) - \frac{2q}{T_h}(\ell-2x)A(\cos\frac{2\pi t}{\tau}-1\Big] \\
&\times\Big[\frac{q}{2T_h}(\ell-2x)+A(\ell-2x)(\cos\frac{2\pi t}{\tau}-1)\Big] \\
&-\Big[T_h+\frac{q^2}{8T_h}(\ell-2x)^2\Big]2A(\cos\frac{2\pi t}{\tau}-1) \\
&+\frac{q^2}{8T}(\ell-2x)^2A(\cos\frac{2\pi t}{\tau}-1\Big] \\
&+m\Big[\frac{2\pi}{\tau}\Big]^2A(x\ell-x^2)\cos\frac{2\pi t}{\tau} \\
&+\epsilon d_c^2\Big[-\frac{3a}{\ell^2}(\ell-2x)^2(\cos\frac{2\pi t}{\tau}-1) \\
&+\frac{q}{2T_h}(\ell-2x)^2A(\cos\frac{2\pi t}{\tau}-1)\Big] \\
&\times\Big[-\frac{q}{T_h}-2a(\cos\frac{2\pi t}{\tau}-1)\Big] \\
&+c\Big[\frac{2\pi}{\tau}A(x\ell-x^2)\sin\frac{2\pi t}{\tau}\Big]^2=0
\end{aligned} \tag{13}$$

to find the magnitude of coefficient A, we demand that Eq.(13) be valid for the point of the cable $x_l=\ell/8$ at the moment of time $t_l=\tau/4$. As a result

$$A=\frac{\epsilon d_c^2\frac{81}{8}\frac{a}{\ell}+\frac{27}{32}\epsilon d_c^2\frac{q^2}{T_h^{\,2}}\ell^2+\frac{27}{64}\frac{q^2}{T_h^2}+2T_h}{\frac{27}{16}\epsilon d_c^2\frac{q}{T_h}\ell^2-c(\frac{2\pi}{\tau})^2(\frac{7\ell^2}{64})^2} - \Big[\Big[\frac{\mathcal{A}}{\mathcal{B}}\Big]^2-\frac{\mathcal{C}}{\mathcal{B}}\Big]^{1/2} \tag{14}$$

where

$$\mathcal{A}=\epsilon d_c^2\frac{81}{8}\frac{a}{\ell}+\frac{27}{32}d_c^2\frac{q^2}{T_h^2}+\frac{27}{64}\frac{q^2}{T_h}\ell^2+2T_h$$

$$\mathcal{B}=\frac{27}{16}\epsilon d_c^2\frac{q}{T_h}\ell^2-c(\frac{2\pi}{\tau})^2(\frac{7\ell^2}{64})^2$$

$$\mathcal{C}=\frac{81}{16}\epsilon d_c^2\frac{q}{T_h}\frac{a}{\ell}$$

The increase in tension due to the movement of the right-hand point is

$$T_g=\epsilon d_c^2(-\frac{3a}{\ell}+A\frac{q}{2T_h}\ell^2)(\cos\frac{2\pi t}{\tau}-1)$$

where A is defined in Eq. (14).

Equation (14) seems to be rather complicated, but by using it, it is possible to analyze the influence of the cable's extensibility, length, and period of movement on tension in the cable, for example, when $\tau\to 0$, $A\to 0$. In this case the cable does not change its form. The movement of the right-hand point is possible only because of the cable's extension. Whenever $\epsilon\to\infty$, we will obtain the formula for an inextensive cable. Many assumptions which were made to obtain Eqs. (14) and (15) are obviously not valid. Consequently, an experiment was carried out to estimate their influence. The scheme of the experiment is shown in Fig. 3. In this experiment the tow-boat's movements and the cable tension were recorded simultaneously. A steel cable 100 m long with $d_c=17.5$mm was used. The difference between the cable tension measured and calculated was 5 to 7% of the maximum magnitude of tension.

Whenever the towing cable is made of two or more sections with different extensibilities, it is necessary to use average extensibility in Eqs. (14) and (15). When the cable consists of two parts, the average extensibility may be calculated using Hooke's law.

$$(\epsilon d_c^2)_{av}=(\ell_1+\ell_2)(\epsilon d_c^2)_1(\epsilon d_c^2)_2/[\ell_1(\epsilon d_c^2)_2+\ell_2(\epsilon d_c^2)_1]$$

where ℓ_i and $(\epsilon d_c^2)_i$ are length and extensibility of the ith part of the cable. This method is not exact, but its accuracy is reasonable enough for approximate calculations.

References

[1] Breslin, J.P., "Dynamic Forces Exerted by Oscillating Cables," *Journal of Hydronautics*, Vol. 8, Jan. 1974, pp. 19-31.

Flow Past an Anchored Slender Ship in Variable-Depth Shallow Water: An Extension

Allen Plotkin*
University of Maryland, College Park, Md.

Introduction

In a previous paper, Plotkin[1] obtained the solution for the steady subcritical potential flow past a slender ship in shallow water in the presence of a slender bump. This work extended the constant-depth results of Tuck[2] by finding the second-order correction to the vertical force and pitching moment acting on the ship. It is the purpose of this Note to demonstrate that the solution technique of Ref. 1 is applicable to more general depth variations, and that changes in depth of the order of the slenderness parameter can lead to contributions to the forces of comparable magnitude to the constant-depth results.

Received May 28, 1975. This research was supported by the National Science Foundation under Grant ENG 74-20573 and by the Minta Martin Fund of the University of Maryland.
Index category: Hydrodynamics.
*Associate Professor of Aerospace Engineering. Member AIAA.